

VII. *Remarks on a correction of the solar tables required by Mr. SOUTH's observations.* By G. B. AIRY, Esq. M. A. Fellow of Trinity College, Cambridge, and Lucasian Professor of Mathematics in the University of Cambridge. Communicated by Dr. YOUNG, F. R. S. &c.

Read February 15, 1827.

THE discordances between the sun's true \mathcal{R} , as observed by Mr. SOUTH, and his calculated \mathcal{R} , as given in the Nautical Almanac, follow a law so simple, as to justify us in attributing them principally to the errors of the solar tables. The only exceptions to this assertion are the differences of March 1st, December 22d, and December 23d, 1822; and upon examination of the sun's calculated \mathcal{R} for several days, previous and subsequent to those days, it appears that there is some irregularity in the second differences. I imagine therefore that some small errors have been accumulated in the calculations for those days; but as this is merely conjectural, I have not thought myself at liberty to reject them in the following computations.

A single inspection of the discrepancies, is sufficient to show that they are almost entirely produced by an error of the epoch, and an error in the place of the perigee. With these errors only I begun my calculations; but finding that the construction of the solar tables (contained in VINCE's Astronomy) gave great facilities for introducing an error in the eccentricity, I begun my calculations again, supposing the epoch, the place of the perigee, and the eccentricity, or the greatest equation

MDCCCXXVII.

K

of the centre, liable to error. The error of the equation of the centre is found to be so small that it may be neglected: but the errors of the epoch and place of the perigee are considerable.

The first part of the operation was to deduce from the errors in \mathbf{R} , the corresponding errors in longitude. This was done by multiplying them by $15 \text{ sec. } 23^\circ 28' \cos.^2 \text{ dec.}$; the multiplication by $\text{sec. } 23^\circ 28'$ was however reserved to the end, when the results are multiplied by it. The next was, to give the errors which would be occasioned by assumed errors in the epoch, the place of the perigee, and the greatest equation of the centre. As the tables contain the variation of the equation of the centre for a variation of $10'$ in the mean anomaly, and for one of $-17'',18$ in the greatest equation, this was very easily effected.

Supposing then that the epoch ought to be increased by x'' , the mean anomaly by $y \times 10'$, and the greatest equation of the centre by $-z \times 17'',18$, I get the following equations, each of which is erroneous to the amount of the error of observation. The first side (as was mentioned) ought to be multiplied by $\text{sec. } 23^\circ 28'$.

$$\begin{aligned}
 10,57 &= x - y \times 19,7 - z \times 0,48 \\
 11,87 &= x - y \times 19,6 + z \times 2,1 \\
 10,38 &= x - y \times 19,4 + z \times 2,91 \\
 13,87 &= x - y \times 19,1 + z \times 4,59 \\
 12,8 &= x - y \times 18,9 + z \times 4,87 \\
 10,31 &= x - y \times 16,9 + z \times 8,75 \\
 8,94 &= x - y \times 16,7 + z \times 8,99 \\
 12,49 &= x - y \times 15,6 + z \times 10,43 \\
 10,76 &= x - y \times 15,4 + z \times 10,65 \\
 8,43 &= x - y \times 14 + z \times 12
 \end{aligned}$$

$$\begin{aligned}
 14,16 &= x - y \times 13,4 + z \times 12,54 \\
 10,51 &= x - y \times 13,1 + z \times 12,74 \\
 10,85 &= x - y \times 12,8 + z \times 12,93 \\
 10,85 &= x - y \times 12,6 + z \times 13,12 \\
 9,81 &= x - y \times 12,3 + z \times 13,31 \\
 9,73 &= x - y \times 9,9 + z \times 14,79 \\
 9,90 &= x - y \times 9,6 + z \times 14,94 \\
 9,76 &= x - y \times 9,3 + z \times 15,08 \\
 9,76 &= x - y \times 9 + z \times 15,22 \\
 11,99 &= x - y \times 6,8 + z \times 16,05 \\
 10,19 &= x - y \times 5,8 + z \times 16,34 \\
 12,09 &= x - y \times 5,5 + z \times 16,43 \\
 11,6 &= x - y \times 2,8 + z \times 16,96 \\
 10,52 &= x - y \times 2,5 + z \times 17, \\
 11,93 &= x - y \times 0,1 + z \times 17,18 \\
 10,57 &= x + y \times 6,5 + z \times 16,38 \\
 9,25 &= x + y \times 6,8 + z \times 16,3 \\
 7,51 &= x + y \times 9,1 + z \times 15,51 \\
 8,6 &= x + y \times 9,4 + z \times 15,37 \\
 7,85 &= x + y \times 11,5 + z \times 14,32
 \end{aligned}$$

$$\begin{aligned}
 7,43 &= x + y \times 17,9 + z \times 8,59 \\
 8,01 &= x + y \times 18,3 + z \times 8,05 \\
 5,43 &= x + y \times 18,4 + z \times 7,77 \\
 7,22 &= x + y \times 18,5 + z \times 7,5 \\
 5,78 &= x + y \times 18,9 + z \times 6,98 \\
 6,39 &= x + y \times 19,3 + z \times 6,11 \\
 6,21 &= x + y \times 19,8 - z \times 4,53 \\
 6,54 &= x + y \times 19,8 - z \times 4,78
 \end{aligned}$$

$$\begin{aligned}
 8,74 &= x + y \times 12,6 - z \times 13,7 \\
 5,52 &= x + y \times 12 - z \times 14,07 \\
 8,45 &= x + y \times 11,7 - z \times 14,24
 \end{aligned}$$

$$\begin{aligned}
 3,31 &= x + y \times 10,6 - z \times 14,87 \\
 0,44 &= x + y \times 10,3 - z \times 15,01 \\
 10,04 &= x - y \times 9,7 - z \times 14,87 \\
 9,87 &= x - y \times 10 - z \times 14,72
 \end{aligned}$$

$$\begin{aligned}
 11,39 &= x - y \times 15,1 - z \times 10,99 \\
 12,19 &= x - y \times 15,3 - z \times 10,77 \\
 7,84 &= x - y \times 15,7 - z \times 10,31 \\
 12,81 &= x - y \times 16,3 - z \times 9,6 \\
 12,54 &= x - y \times 17 - z \times 8,62 \\
 10,65 &= x - y \times 17,1 - z \times 8,41 \\
 10,62 &= x - y \times 17,3 - z \times 8,16 \\
 11,22 &= x - y \times 17,5 - z \times 7,9 \\
 9,02 &= x - y \times 17,6 - z \times 7,64 \\
 11,85 &= x - y \times 17,9 - z \times 7,12 \\
 9 &= x - y \times 18 - z \times 6,86 \\
 8,76 &= x - y \times 19,5 - z \times 2,8 \\
 11,19 &= x - y \times 19,7 + z \times 0,6 \\
 13,99 &= x - y \times 19,6 + z \times 1,44 \\
 12,51 &= x - y \times 19,6 + z \times 2,29 \\
 12,01 &= x - y \times 17,5 + z \times 7,94 \\
 9,08 &= x - y \times 17,3 + z \times 8,2 \\
 11,03 &= x - y \times 17,2 + z \times 8,45 \\
 10,03 &= x - y \times 17 + z \times 8,7 \\
 9,53 &= x - y \times 16,8 + z \times 8,95 \\
 11,53 &= x - y \times 16 + z \times 9,88 \\
 11,98 &= x - y \times 15,8 + z \times 10,15 \\
 9,43 &= x - y \times 14,1 + z \times 11,9
 \end{aligned}$$

$$\begin{aligned}
 10,57 &= x - y \times 13,9 + z \times 12,1 \\
 8,03 &= x - y \times 13,6 + z \times 12,31 \\
 8,25 &= x - y \times 13,2 + z \times 12,68 \\
 9,31 &= x + y \times 4 + z \times 16,9
 \end{aligned}$$

$$\begin{aligned}
 6 &= x + y \times 10,9 + z \times 14,69 \\
 5,05 &= x + y \times 12,3 + z \times 13,86 \\
 4,65 &= x + y \times 13,5 + z \times 13,09 \\
 5,85 &= x + y \times 13,7 + z \times 12,89. \\
 \hline
 4 &= x + y \times 16,7 + z \times 10,2 \\
 6,07 &= x + y \times 17,3 + z \times 9,43 \\
 6,96 &= x + y \times 18,5 + z \times 7,59 \\
 6,72 &= x + y \times 18,7 + z \times 7,31 \\
 5,05 &= x + y \times 18,8 + z \times 7,02 \\
 2,07 &= x + y \times 20,3 + z \times 2,9 \\
 2 &= x + y \times 20,4 + z \times 2,6 \\
 4,7 &= x + y \times 20,5 + z \times 1,72 \\
 3,53 &= x + y \times 20,5 + z \times 1,09 \\
 6,23 &= x + y \times 20,5 + z \times 0,47
 \end{aligned}$$

Grouping together those equations in which the sun's anomaly is included between $1^{\circ} 30'$ and $4^{\circ} 30'$, between $4^{\circ} 30'$ and $7^{\circ} 30'$, between $7^{\circ} 30'$ and $10^{\circ} 30'$, and between $10^{\circ} 30'$ and $1^{\circ} 30'$, as marked by the divisions above, we have the following results.

Summer of 1821.

$$110,42 = 10x - y \times 175,3 + z \times 64,81.$$

Autumn of 1821.

$$207,43 = 20x - y \times 82,2 + z \times 302,51.$$

Winter of 1821-1822.

$$53,01 = 8x + y \times 150,9 + z \times 35,69.$$

Spring of 1822.

$$46,37 = 7x + y \times 37,5 - z \times 101,48.$$

Summer of 1822.

$$250,2 = 23x - y \times 394,9 - z \times 20,68.$$

Autumn of 1822.

$$57,71 = 8x + y \times 13,7 + z \times 108,52.$$

Winter of 1822.

$$47,33 = 10x + y \times 192,2 + z \times 50,33.$$

And adding the groups for the same seasons of the two years,

$$\begin{array}{l} \text{Observations} \\ \text{in Spring give} \end{array} \left. \begin{array}{l} 46,37 = 7x + y \times 37,5 - z \times 101,48 \\ \text{in Summer} - 360,62 = 33x - y \times 570,2 + z \times 44,13 \\ \text{in Autumn} - 265,14 = 28x - y \times 68,5 + z \times 411,03 \\ \text{in Winter} - 100,34 = 18x + y \times 343,1 + z \times 86,02 \end{array} \right\}$$

Adding all,

$$772,47 = 86x - y \times 258,1 + z \times 439,7.$$

Subtracting the Winter from the Summer equations,

$$260,28 = 15x - y \times 913,3 - z \times 41,89.$$

Subtracting the Spring from the Autumn equations,

$$218,77 = 21x - y \times 106 + z \times 512,51.$$

By solving these equations we find $x = 8,23$, $y = -\frac{1}{6,555}$, $z = \frac{1}{17,18}$. These are to be multiplied by sec. $23^\circ. 28' = 1,0902$. Performing this multiplication, and forming the quantities x'' , $y \times 10'$, and $-z \times 17''18$ we find that the epoch ought to be increased $8''97$, the mean anomaly ought to be diminished $99''8$, and the greatest equation of the centre diminished $1''09$. The epoch of the perigee ought therefore to be increased $107''8$.

The correction of the equation of the centre is so small, that it may be doubtful whether it would be necessary to consider it. In that case, the solar tables would require no other alteration than in the tables of epochs. Every epoch of the sun must be increased by $8''97$ or $9''$, and every epoch of the perigee by $1'48''$.

If we reject the equation corresponding to the observations of March 1st, December 22d, and December 23d, 1822, (the 43d, 82d, and 83d of the preceding list), we find that the epoch of the sun's longitude must be increased by $9''3$, and that of the perigee by $1'.39''$, and that the greatest equation of the centre must be diminished by $0''66$.